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A Variable Diffusivity Model for the Drying of Spherical Food Particulates

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Abstract. An investigation of the drying of spherical food particles was performed, using peas as the model material. In the development of a mathematical model for drying curves, moisture diffusion was modelled using Fick's second law for mass transfer. The resulting partial differential equation was solved using a forward-time central-space finite difference approximation, with the assumption of variable effective diffusivity. In order to test the model, experimental data was collected for the drying of green peas in a fluidised bed at three drying temperatures. Through fitting three equation types for effective diffusivity to the data, it was found that a linear equation form, in which diffusivity increased with decreasing moisture content, was most appropriate. The final model accurately described the drying curves of the three experimental temperatures, with an R^2 value greater than 98.6% for all temperatures.

Nomenclature

A	coefficient 1 for effective diffusivity equation, m^2/s
B	coefficient 2 for effective diffusivity equation, dependent on equation type
C	coefficient 1 for Arrhenius type equation, m^2/s
D	coefficient 2 for Arrhenius type equation, m^2/s
D_{eff}	effective diffusivity, m^2/s
M	moisture content, kg water/kg dry matter
M_e	equilibrium moisture content, kg water/kg dry matter
M_0	initial moisture content, kg water/kg dry matter
MR	dimensionless moisture ratio
R	gas constant, $JK^{-1}mol^{-1}$
R^2	coefficient of determination, %
r	radius, m
T	temperature, $^{\circ}C$
t	time, s
WMR	dimensionless weighted moisture ratio

Introduction

Drying is a common practice in the food industry and is used to increase shelf life, decrease packaging requirements and reduce shipping weight and volume [1]. The process of drying causes physical, chemical and nutritional changes due to the decrease in moisture content and increased temperatures [2]. In industry, mathematical models are used to predict potential undesired effects and as such, food drying is an area which has been studied extensively. As a result, a large amount of research exists on the modelling of food drying curves.

The common model for the drying of fruits and vegetables makes several assumptions. It is known that during the process of drying simultaneous heat and mass transfer takes place [2]. In order to simplify the resulting model, it is commonly assumed that this process can be treated as isothermal, and thus, that heat transfer can be considered negligible [3]. Other assumptions include that mass transfer occurs symmetrically, that the material is isotropic and that the particulate size and shape remains constant during the drying process [4].

One area of open research, and the focus of this paper, is whether the diffusivity of food material is variable during the drying process and if so, how to accurately model this variability. To date, it has been recognized that there is a relationship between diffusivity and moisture content [5]. This was confirmed by Hatamipour and Mowla in a study of the correlations between shrinkage, density and diffusivity, however there is no confirmed model for the effect of temperature and moisture content on effective diffusivity [6].

A number of studies have been performed in which the variation of effective diffusivity with moisture content has been modelled. In a study of the drying characteristics of shrinkable food products, Ruiz-Lopez et al. found that moisture diffusivity exhibited a complex behaviour, with water diffusivity increasing as moisture fraction decreased until a maximum at a moisture ratio of 0.6 after which it decreased [7]. Hashemi, Mowla and Kazemeini found that during the drying of broad beans, the effective diffusivity increased with increasing moisture, until a point at which it either leveled off or began to decrease [8]. Conversely, Singh, Kumar and Gupta studied the drying of carrot cubes and found that the effective diffusivity decreased with decreasing moisture content [9].

In the creation of drying models, effective diffusivity is commonly treated as a constant value. In their study on the drying of peas, Simal et al. assumed that the effective diffusivity only varied with air temperature [4]. The effective diffusivity values were calculated for different air temperatures using an Arrhenius type equation. This treatment of effective diffusivity is common and, for example, is used in studies by Wang and Brennan, Simal et al. and Orikasa et al. [2,4,10].

Effective diffusivity has also been included in the form of an equation relating to moisture content. In their study on the drying of banana slices, Thuwapanichayanan et al. proposed an equation in which the effective diffusivity decreased exponentially with decreasing moisture content [11].

It is clear from this literature that there is no confirmed model for drying which includes variable effective diffusivity. Furthermore, in literature, studies on drying models for peas are scarce. As such, the purpose of this research is to develop an accurate model for the drying of green peas, which considers the possibility of variable effective diffusivity.

Mathematical Model

For the model, it was assumed that heat transfer is negligible and therefore, only mass transfer was considered. Using Fick's second law, as applied to a sphere, the required equation for the rate of change of moisture with time can be derived as shown in equation 1 [12].

$$\frac{\partial M}{\partial t} = D_{eff} \left(\frac{\partial^2 M}{\partial r^2} + \frac{2}{r} \frac{\partial M}{\partial r} \right) \quad (1)$$

Assuming that the material is isotropic and the initial moisture content is uniform throughout the solid, the initial and boundary conditions for the solution of this equation are given below:

$$t = 0, 0 < r < R, \mathbf{M}(r, 0) = M_0 \quad (2)$$

$$t > 0, r = 0, \frac{\partial M}{\partial r} = 0 \quad (3)$$

$$t > 0, r = R, \mathbf{M}(R, t) = M_e \quad (4)$$

In order to solve this equation, the finite difference method was selected, as it provides a flexible solution method for this type of equation. In order to apply the finite difference method, the geometry must be divided into a discrete grid in time and space. For a sphere, there is one dimension in space, r . Therefore, the grid is a series of concentric shells, of constant thickness Δr , except for the outermost shell, which has a thickness of $\Delta r/2$. This allows the boundary condition for the surface to be applied to the outer shell.

Using the forward difference approximation for the first order terms and the central difference equation for the second order term, the finite difference approximation of each term in equation 1 can be written as follows:

$$\frac{\partial M}{\partial t} = \frac{M(r, t+\Delta t) - M(r, t)}{\Delta t} \quad (5)$$

$$\frac{\partial M}{\partial r} = \frac{M(r+\Delta r, t) - M(r, t)}{\Delta r} \quad (6)$$

$$\frac{\partial^2 M}{\partial r^2} = \frac{M(r+\Delta r, t) + M(r-\Delta r, t) - 2M(r, t)}{\Delta r^2} \quad (7)$$

These equations were substituted into Equation 1 and rearranged to develop an equation for the moisture content for the following time step, $M(r, t+\Delta t)$. This type of rearrangement is known as a forward-time central-space (FTCS) scheme [13]. The solution for this equation will give the moisture content of each shell for each time step.

For comparison of results, the moisture content at each time step was expressed as the moisture ratio, **MR**. The moisture ratio is a dimensionless ratio between the moisture content, initial moisture content and the equilibrium moisture content, which can be calculated according to Equation 8.

$$MR(r, t) = \frac{M(r, t) - M_e}{M_0 - M_e} \quad (8)$$

In order to compare the theoretical results with the experimental results, an average moisture ratio (**WMR**(t)) must be computed, such that the change in moisture over time can be represented by a single curve. This was calculated by weighting the moisture ratio of each shell based on relative volume.

The solution of this model was performed using a program written in MATLAB. This program used an iterative process to calculate the moisture content at each shell for each time step and then produced a single curve of average moisture ratio against drying time.

Data Collection

In order to determine an appropriate equation for effective diffusivity and to judge the accuracy of the model, data was collected for the drying of green peas at three drying temperatures (30°C, 40°C and 50°C). Fresh green peas (*Pisum sativum*) of the Bounty variety were used for this experiment. Peas with an average diameter of 10 ± 1 mm were selected and stored in a cold room at 4°C for 24 hours before experimentation to reach stable moisture content. The peas were dried using a fluidised bed dryer, as shown in Figure 1, which was connected to a heat pump dehumidifier system.

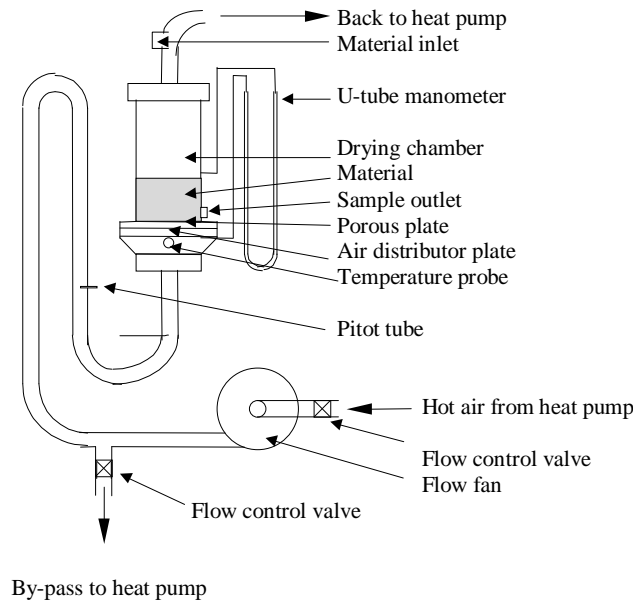


Figure 1. Fluidised bed dryer

The drying temperature was set by the temperature controller in the heat pump dehumidifier system and the velocity of the hot air was set to 2.2 m/s. During the experiment, samples were taken at 30 minute intervals via the sample outlet for measurement. The moisture content of the samples was determined using a vacuum oven in accordance with AOAC method 934.06 (AOAC International 1995).

Results and Discussion

Effective Diffusivity. Three equation types for the effective diffusivity term, D_{eff} , in the mathematical model, were considered: constant ($D_{eff}=A$), linear ($D_{eff}=A(1-WMR(t))$) and exponential ($D_{eff}=Ae^{-B(WMR(t))}$). For each equation type, the least squares error method was used to find the coefficient value(s) which produced the model closest to the experimental data. In order to determine the most appropriate equation type, the best model produced using each type was compared against the experimental results, as shown in Figure 2. The least squares error for each model type, when compared to the experimental data, is given in Table 1.

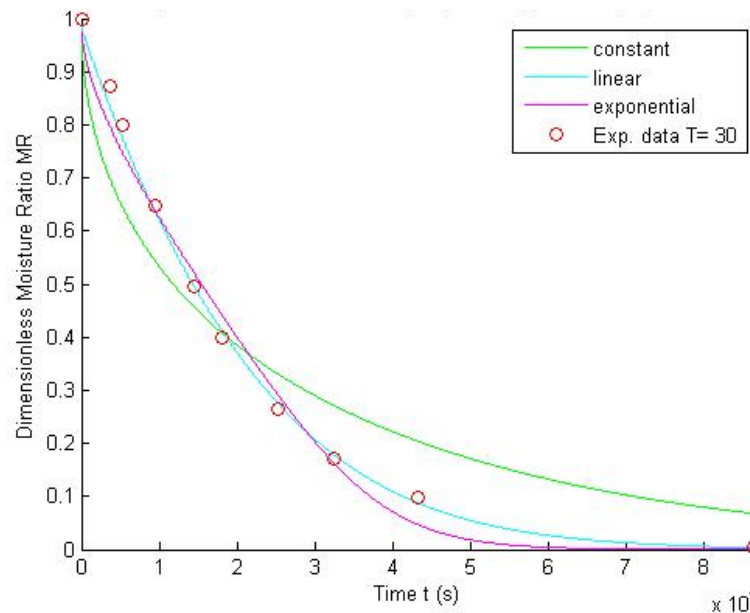


Figure 2. Comparison of three models for effective diffusivity for drying of peas at 30°C

Table 1. Comparison of least squares error for three models for effective diffusivity at 30°C

Equation Type	Least Squares Error
Constant	0.00969
Linear	0.000357
Exponential	0.00143

It can be seen in Figure 2 and Table 1 that the linear equation form for effective diffusivity produced the model with the lowest error. This was true for all temperatures. The least squares error for this equation type was, as a minimum, four times less than the error of the constant and exponential models, across all temperatures. Therefore, the linear equation form will be used for the final model.

In order for the final model to predict the drying curve for any experimental temperature, it is necessary to be able to calculate the effective diffusivity constant, A , using temperature only. This was achieved using an Arrhenius type equation, which is shown in equation 9.

$$A = Ce^{-\frac{D}{RT}} \quad (9)$$

The best values for the Arrhenius type equation coefficients, C and D , were determined by fitment to the experimental data using a least squares method. The resulting values are given in Table 2.

Table 2. Arrhenius type equation coefficients

Coefficient	Value
C	$0.0420 \text{ m}^2/\text{s}$
D	47.7 kJ/mol

Final Model. In summary, it was determined that effective diffusivity follows a linear trend with respect to moisture ratio. This was parameterized by a single coefficient, A . It was determined that the value of A increases exponentially with increasing temperature and was described by an Arrhenius type exponential equation with two coefficients, C and D . The drying curves predicted by the proposed mathematical model, as compared with the experimental curves, can be seen in Figure 3.

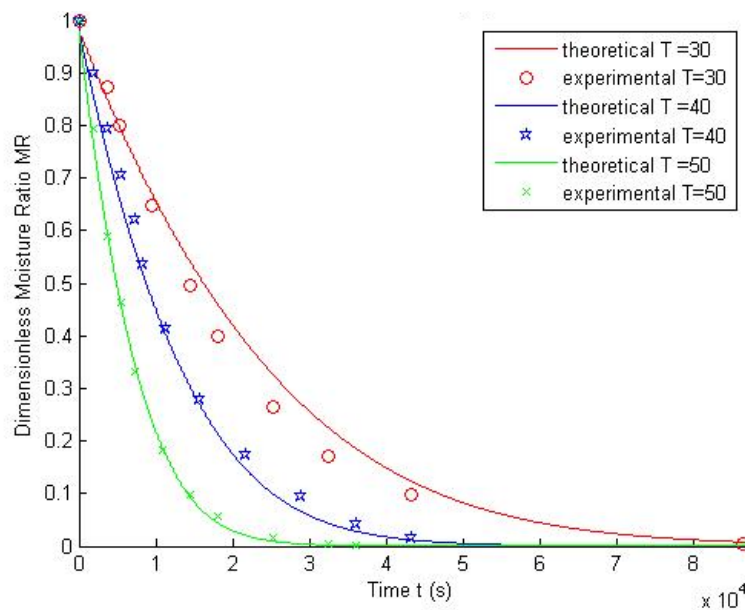


Figure 3. Experimental and computed dimensionless moisture content

The effectiveness of the model in predicting the experimental drying curves can be calculated using the coefficient of determination, also known as the R^2 value. This type of error measurement is a measure of the proportion of variability in the data set that is accounted for by the theoretical model, with an R^2 value of 100% corresponding to a perfect model. The corresponding R^2 values for each curve in Figure 3 are given in Table 3.

Table 3. Coefficient of determination comparing predicted and experimental results

Temperature [°C]	R^2 [%]
30	98.6
40	99.2
50	99.9

Discussion. It can be seen in Figure 3 and Table 3 that the proposed model provides a fairly accurate prediction of the drying curves for spherical foods in a fluidized bed. By comparing the errors for the three equation types for effective diffusivity, it was seen that the error for the linear model is significantly less than the error for the constant and exponential models. From these results, it is clear that the variability of effective diffusivity is not negligible for the drying of peas.

In order to confirm this model it would be necessary to compare the theoretical model with the drying curve for a temperature that was not used in the creation of the model. However this additional data was not available at the time of study.

Extensions to this model could include application to other food materials and other food geometries. The results of these studies would verify this form of model creation.

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